

Pinning down the New Minimal Supersymmetric GUT

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ABSTRACT

We show that generic $\mathbf{10} \oplus \mathbf{120} \oplus \overline{\mathbf{126}}$ fits of fermion masses and mixings, using real superpotential couplings but with complex ‘Higgs fractions’ leading to complex yukawa couplings in the effective MSSM, *overdetermine*(by one extra constraint) the superpotential parameters of the new Minimal Supersymmetric SO(10) GUT[1]. Therefore fits should properly be done by generating the 24 generic fit parameters from the 23 parameters of the NMSGUT superpotential, given $\tan\beta$ as input. Each numerical fit then *fully specifies* the parameters of the NMSGUT. An analysis of all its implications, modulo only the residual uncertainty of supersymmetry breaking parameters, is now feasible. Thus the NMSGUT offers the possibility of a confrontation between the scale of gauge unification and the fit to fermion masses due to their extractable common dependence on the NMSGUT parameters. If and when ‘smoking gun’ discoveries of Supersymmetry and Proton decay occur they will find the NMSGUT fully vulnerable to falsification.

1 Introduction

A series of papers over the last few years[2, 3, 4, 5, 7, 1, 8] have developed the renormalizable Supersymmetric SO(10) GUT based on the Higgs set $\mathbf{210} \oplus \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \oplus \overline{\mathbf{126}}$ (the so called New Minimal Supersymmetric GUT(NMSGUT)) into a theory capable of encompassing the entire gamut of fermion mass-mixing data in a most parameter economical way, while preserving the traditional advantages and attractions of renormalizable Supersymmetric SO(10)GUTs[9, 10, 11, 12, 13, 14, 15]. The NMSGUT theory has only 23 superpotential parameters and one gauge coupling among its ‘hard’ parameters. This is one less parameter than the original MSGUT[9, 10, 12] in spite of the introduction of the $\mathbf{120}$ representation to save the feasibility of the fermion fit [2, 3, 4, 5, 1, 6]. It also has a very characteristic pattern of fermion yukawas where the $\mathbf{10}, \mathbf{120}$ couplings to $\mathbf{16} \cdot \mathbf{16}$ must necessarily dominate[3, 4, 5, 7, 8, 1, 6] those of the $\overline{\mathbf{126}}$ to permit a (Type I) seesaw mechanism to generate the observed neutrino masses. This domination also results in right handed neutrinos that are lighter than the GUT scale : which may be of importance for cosmology.

The work of [7, 8] has determined accurate “generic” numerical fits of the fermion mass data by following the strategy of coupling domination mentioned above and moreover restricting attention to the case of real superpotential couplings i.e only spontaneous CP violation. Although prima facie gratifying, these successes, like those of the generic fits without the $\mathbf{120}$ [16], survive only with angst[7, 8] concerning their realization in the full NMSGUT. In this paper we show that this angst is fully justified. The generic parametrization assumes a freedom that it is not generically entitled to because the underlying structure of the NMSGUT in fact imposes one constraint among the generic parameters which they will not, in general, satisfy. One finds that 11 of the generic parameters[7] may be expressed in terms of just 10 NMSGUT dimensionless parameters. Therefore the numerical fitting program must be carried out while respecting one constraint. The only feasible way of doing this is to take the NMSGUT parameters as the free fitting variables instead of the generic ones. Thus each fit will lead to NMSGUT parameters sets that fully specify the theory modulo supersymmetry breaking uncertainties. Indeed, in spite of the incomplete Lepton data, the fitting exercise will still yield complete and prima facie consistent parameter sets which can then be processed to make the latent internal contradictions, e.g. those between the fermion data fit and the gauge RG flow, or those arising due to the use of the still incomplete fermion data, emerge. Thus even before supersymmetry or proton decay are observed we may obtain significant insights into the entire structure of the NMSGUT ! In this letter we give only the analytic details of the above arguments and leave the very involved numerical implementation to succeeding works[19].

2 Basics of NMSGUT

The NMSGUT [1] is a renormalizable globally supersymmetric $SO(10)$ GUT whose Higgs chiral supermultiplets consist of AM(Adjoint Multiplet) type totally antisymmetric tensors : $\mathbf{210}(\Phi_{ijkl})$, $\overline{\mathbf{126}}(\overline{\Sigma}_{ijklm})$, $\mathbf{126}(\Sigma_{ijklm})(i, j = 1...10)$ which break the GUT symmetry to the MSSM, together with Fermion mass (FM) Higgs $\mathbf{10}(\mathbf{H}_i)$ and $\mathbf{120}(O_{ijk})$. The $\overline{\mathbf{126}}$ plays a dual or AM-FM role since it also enables the generation of realistic charged fermion and neutrino masses and mixings (via the Type I and/or Type II Seesaw mechanisms); three $\mathbf{16}$ -plets $\Psi_A(A = 1, 2, 3)$ contain the matter including the three conjugate neutrinos ($\bar{\nu}_L^A$). The superpotential (see[12, 13, 14, 15, 1] for comprehensive details) contains the mass parameters

$$m : \mathbf{210}^2 \quad ; \quad M : \mathbf{126} \cdot \overline{\mathbf{126}}; \quad M_H : \mathbf{10}^2; \quad m_O : \mathbf{120}^2 \quad (1)$$

and trilinear couplings

$$\begin{aligned} \lambda : \mathbf{210}^3 \quad ; \quad \eta : \mathbf{210} \cdot \mathbf{126} \cdot \overline{\mathbf{126}}; \quad \gamma \oplus \bar{\gamma} : \mathbf{10} \cdot \mathbf{210} \cdot (\mathbf{126} \oplus \overline{\mathbf{126}}) \\ k : \mathbf{10} \cdot \mathbf{120} \cdot \mathbf{210} \quad ; \quad \rho : \mathbf{120} \cdot \mathbf{120} \cdot \mathbf{210} \\ \zeta : \mathbf{120} \cdot \mathbf{210} \cdot \mathbf{126} \quad ; \quad \bar{\zeta} : \mathbf{120} \cdot \mathbf{210} \cdot \overline{\mathbf{126}} \end{aligned} \quad (2)$$

In addition one has two symmetric matrices h_{AB}, f_{AB} of Yukawa couplings of the the $\mathbf{10}, \overline{\mathbf{126}}$ Higgs multiplets to the $\mathbf{16.16}$ matter bilinears and one antisymmetric matrix g_{AB} for the coupling of the $\mathbf{120}$ to $\mathbf{16.16}$. It was shown[3, 4, 1, 7] that with only spontaneous CP violation, i.e with all the superpotential parameters real, it is till possible to achieve an accurate fit of all the fermion mass data which furthermore evades the difficulties encountered in accommodation with the high scale structure of the MSGUT[3] provided[4, 5, 1, 7, 8, 6] one takes the $\mathbf{10}, \mathbf{120}$ yukawa couplings to be much larger than those of the $\overline{\mathbf{126}}$ so that Type I neutrino masses are enhanced. However in this letter we point out that the numerical nonlinear fitting procedure[7] must respect a further constraint so that the use of NMSGUT parameters becomes mandatory.

The GUT scale vevs and therefore the mass spectrum are all expressible in terms of a single complex parameter x which is a solution of the cubic equation

$$8x^3 - 15x^2 + 14x - 3 = -\xi(1 - x)^2 \quad (3)$$

where $\xi = \frac{\lambda M}{\eta m}$.

Spontaneous CP violation implies that x must lie[1] on one of the two complex solution branches $x_{\pm}(\xi)$, ($\xi \in (-27.917, \infty)$). Since λ, η are already counted as independent $x_+(\xi)$ counts for M/m .

3 NMSGUT Constraints on the Grimus Kühböck generic parametrization

The generic parametrization[7] of fermion masses, in terms of the Yukawa-vev products and dimensionless parameters arising from doublet vev ratios and vev phases, can be translated in terms of the “Higgs fractions” $(\alpha_i, \bar{\alpha}_i)$ determined by the fine tuning that keeps the MSSM pair of Higgs doublets light[12, 13, 14, 15, 1]. We show that 12 dimensionless parameters of the generic fit $(\xi_{u,d,l,D}, \zeta_{u,d}, r_{F,H,u,l,D,R})$ are determined in terms of only 11 superpotential couplings $(m_O/m, M/m, m/v, \eta, \lambda, \zeta, \bar{\zeta}, \rho, \gamma, \bar{\gamma}, k)$ of the NMSGUT (M_H is fixed by finetuning). Note that even the GUT scale parameter (m) that sets the mass scale of all superheavy particles[12, 13, 15, 14, 1] is determined by the Type I seesaw fit. The freedom to choose 12 real matter fermion Yukawas are of course common to both.

To see how these relations arise we need only compare the fermion mass formulae of the generic parametrization[7] with those given by us earlier for the MSGUT[15] and the NMSGUT[3, 1]. The generic parametrization reads

$$M_d = H' + e^{i\xi_d} G' + e^{i\zeta_d} F', \quad (4)$$

$$M_u = r_H H' + r_u e^{i\xi_u} G' + r_F e^{i\zeta_u} F', \quad (5)$$

$$M_\ell = H' + r_\ell e^{i\xi_\ell} G' - 3 e^{i\zeta_d} F', \quad (6)$$

$$M_D = r_H H' + r_D e^{i\xi_D} G' - 3 r_F e^{i\zeta_u} F', \quad (7)$$

$$M_\nu = r_R M_D F'^{-1} M_D^T. \quad (8)$$

The ratios $r_{H,F,D,R,u,l}$ are real by definition since they extracted[7] the phases from the VEVs. On the other hand we had previously given explicit formulae for all light fermion masses in terms of the fundamental (N)MSGUT parameters[15, 3, 1]. These formulae are expressed in terms of the so called Higgs fractions $\alpha_i, \bar{\alpha}_i; i = 1..6$ which specify[1] the MSSM Higgs multiplet pair $H = H^{(1)}, \bar{H} = \bar{H}^{(1)}$ as a linear combination of the 6 pairs of multiplets $h_i[1, 2, 1], \bar{h}_i[1, 2, -1]$ present among the GUT Higgs fields :

$$H = \sum_{i=1}^{i=6} \alpha_i^* h_i \quad ; \quad \bar{H} = \sum_{i=1}^{i=6} \bar{\alpha}_i^* \bar{h}_i \quad (9)$$

The overall phase of the $\alpha_i, \bar{\alpha}_i$ is arbitrary so that we can fix $\alpha_1, \bar{\alpha}_1$ to be real by a choice of this phase to accord with the phase choice in the coefficients of [7]. This modifies the normalization given in[1]. Explicit formulae for the Higgs fractions $\hat{\alpha}_i, \hat{\bar{\alpha}}_i$ in terms of the 9 dimensionless couplings $\tilde{m}_O, \eta, \lambda, \zeta, \bar{\zeta}, \rho, \gamma, \bar{\gamma}, k$ and the parameter x which specifies the GUT scale symmetry breaking $Susy \times SO(10) \rightarrow MSSM$ are given in gory detail in Appendix C of [1]. Just to illustrate the the complexity of the formulae we give the explicit form of the *simplest* of the un-normalized (i.e before

imposing the unitarity constraint $\sum |\alpha_i|^2 = \sum |N\hat{\alpha}_i|^2 = 1 = \sum |\bar{N}\hat{\bar{\alpha}}_i|^2$ coefficients $\hat{\alpha}_1 = \hat{\bar{\alpha}}_1$:

$$\begin{aligned}\hat{\alpha}_1 &= \hat{\bar{\alpha}}_1 = (\tilde{m}_o^2 \eta^2 \lambda P_0 + \bar{\zeta}^2 \zeta^2 \lambda P_1 + \tilde{m}_o \bar{\zeta} \zeta \eta \lambda P_2 + \bar{\zeta} \zeta \eta \lambda \rho P_3 + \tilde{m}_o \eta^2 \lambda \rho P_4 + \eta^2 \lambda \rho^2 P_5) \\ P_0 &= -12 p_3 p_5 t_{(1,1)}^4 & P_1 &= 24 x^3 t_{(1,1)} t_{(10,1)} & P_2 &= -24 x t_{(10,2)} t_{(1,1)}^2 \\ P_3 &= 4 x t_{(11,1)} t_{(1,1)}^2 & P_4 &= 8 p_3 p_5 t_{(2,3)} t_{(1,1)}^3 & P_5 &= 4 x^2 p_3 p_5 t_{(1,1)}^4\end{aligned}$$

where $t_{(m,n)}$ are polynomials in x of degree up to 12. The other $\alpha_i, \bar{\alpha}_i$ are even more complicated !

Then it is straightforward to verify that the equations relating[1] the two parametrizations can be cast in the form ($A, B = 1, 2, 3$ are generation indices) :

$$\begin{aligned}H'_{AB} &= 2\sqrt{2}v\bar{\alpha}_1 \cos \beta h_{AB} \\ F'_{AB} &= 4\sqrt{\frac{2}{3}}v\bar{\alpha}_2 \cos \beta f_{AB} \\ G'_{AB} &= 2\sqrt{\frac{2}{3}}v|i\sqrt{3}\bar{\alpha}_5 + \bar{\alpha}_6| \cos \beta g_{AB}\end{aligned}\tag{10}$$

for the flavour mass matrices,

$$\begin{aligned}\zeta_u &= Arg[\alpha_2] - \frac{\pi}{2} \quad ; \quad \zeta_d = Arg[\bar{\alpha}_2] - \frac{\pi}{2} \\ \xi_u &= Arg[-\sqrt{3}\alpha_5 + i\alpha_6] \quad ; \quad \xi_d = Arg[-\sqrt{3}\bar{\alpha}_5 + i\bar{\alpha}_6] \\ \xi_l &= Arg[-\sqrt{3}\bar{\alpha}_5 - 3i\bar{\alpha}_6] \quad ; \quad \xi_D = Arg[-\sqrt{3}\alpha_5 - 3i\alpha_6]\end{aligned}\tag{11}$$

for the phases, and

$$\begin{aligned}r_H &= \frac{\alpha_1}{\bar{\alpha}_1} \tan \beta \quad ; \quad r_F = \left| \frac{\alpha_2}{\bar{\alpha}_2} \right| \tan \beta \\ r_u &= \left| \frac{\sqrt{3}\alpha_5 - i\alpha_6}{\sqrt{3}\bar{\alpha}_5 - i\bar{\alpha}_6} \right| \tan \beta \\ r_l &= \left| \frac{\sqrt{3}\bar{\alpha}_5 + 3i\bar{\alpha}_6}{\sqrt{3}\alpha_5 - i\alpha_6} \right| \tan \beta \\ r_D &= \left| \frac{\sqrt{3}\alpha_5 + 3i\alpha_6}{\sqrt{3}\bar{\alpha}_5 - i\bar{\alpha}_6} \right| \tan \beta \\ r_R &= \frac{|\bar{\alpha}_2| \lambda \cos \beta}{2\sqrt{3}|\tilde{\sigma}|} \frac{v}{m}\end{aligned}\tag{12}$$

for the ratios of vevs in the parametrization of [7]. Here $\tilde{\sigma}$ is the dimensionless $\overline{126}$ vev in units of m/λ . Note that in eqns(10,11,12) the values of $v, \tan \beta$ (or equivalently

$v_u = v \sin \beta, v_d = v \cos \beta$) are the renormalized values at the GUT scale obtained from RG flow. To two loops the RG equations[17] for the couplings and vevs depend only on the gauge and matter Yukawa couplings together with the input initial value of $\tan \beta$. This is what insulates the deductions from the superpotential parameter set from the depredations of the chaos inducing ignorance regarding supersymmetry breaking parameters, making possible the ambitions of the fitting program.

The first eleven of the eqns(10,11,12) fix 11 of the coefficients of the parametrization in [7] in terms of 10 dimensionless GUT parameters *and hence must obey one constraint*. Then it follows that a fermion fit found (e.g via the downhill simplex method[7, 8]) by freely varying the 12 parameters $\xi_{u,d,l,D}, \zeta_{u,d}, r_{F,H,u,l,D,R}$ and the flavour mass matrices H', F', G' (12 parameters), *must be checked after the fit is achieved to verify whether the constraint is satisfied*. However this is easier said than done because the equations in terms of the GUT parameters are so hopelessly nonlinear[1]. Moreover it is clear that *generically* there is no reason to expect that the constraint will be satisfied. *Therefore the correct procedure to determine fermion fits that do not fall foul of the necessity to respect the constraint imposed by the NMSGUT is obviously to take the NMSGUT superpotential parameters as the freely variable ones*. This is the main conclusion of this letter. The actual implementation of the very lengthy numerical codes in terms of the GUT parameters is currently being carried out and will be reported separately[19]. The reason for the extra computational burden is simply the highly complicated expressions[1] for the Higgs fractions $\alpha_i, \bar{\alpha}_i$ in terms of the GUT parameters. In the next section we conclude with a discussion of where we expect such a modified and consistent fitting program to leave us vis a vis the NMSGUT.

4 Discussion

In this letter we pointed out the correct procedure for fitting the only data currently available for constraining any GUT beyond the basic requirements of gauge unification : the low energy fermion masses and mixing. Although the NMSGUT has 23 superpotential parameters(after the fine tuning to keep a Higgs pair light) *all* of which enter the fermion mass formulae, the number of data values is smaller in number. In the most favourable situation one would have knowledge of 12 fermion masses, 4 CKM parameters, 6 PMNS parameters i.e 22 data in all. Unfortunately, however, of these 22 there is no prospect in sight for measuring the two Majorana phases at present. The leptonic mixing angle θ_{13}^l , the Dirac PMNS phase δ_ℓ and the combination of the neutrino masses effective in neutrinoless double beta decay may become available in the near future and may, in any case, be plausibly bounded from above. Moreover an estimate of neutrino masses may also emerge from cosmology. Practically speaking one must then fit to 11 fermion masses(since only the neutrino mass squared differences are known) and 7 mixing data. Thus -for the present- one

needs to fit 18 data values with 23 parameters through highly nonlinear relations. The only practicable way of doing this is through a numerical fitting procedure such as the downhill simplex algorithm[7, 8] or possibly using some hybrid procedure of an ansatz regarding the hierarchy structure combined with numerical fitting[4, 5]. The accurate generic fits found so far[7, 8] cannot be accommodated in the NMSGUT unless they happen to satisfy the constraint imposed by the NMSGUT ; but that is very difficult -and not really worthwhile- to verify. Instead one should perform the fitting by using the NMSGUT parameters : which are truly independent. Thus what emerges -as has been argued here- is not just a set of plausible values for the generic fitting parameters(for each assumed value of $\tan \beta$) but rather a complete candidate parameter set for the 23 NMSGUT parameters.

The very ‘ease’ of the numerical solution has however a high price, in that one has practically no understanding of the global structure of the solution space. Moreover, since one will perform the fit without constraining the parameters (such as leptonic Majorana phases) whose value one does not know at present, it follows that a variety of fits corresponding to different resultant values of such parameters may in fact be possible. In addition the variation induced by different choices of $\tan \beta$, as well as the RG amplified uncertainties in the fermion data, must also be considered.

However the blindness of the fitting procedure -if successful- opens an interesting possibility : since all GUT parameters are determined by the fermion fit alone we may confront them with the accurate data on gauge couplings at low energy by supplying these fermion data determined values to the RG flow equations for gauge coupling unification. The consistency between these independent determinations of the unification parameters will then serve as an important check on the plausibility of the theory and may even limit the viable values of $\tan \beta$.

The complete determination of the GUT parameters by the fermion fit will also make possible an explicit evaluation of the proton decay rate. Explicit formulae for the effective Superpotential controlling proton decay in terms of the NMSGUT matter fermion yukawa couplings and the Higgs fractions have been derived in [13, 15, 1]. Of course the rate calculation will still be afflicted by the usual superpartner mass and mixing uncertainties[18] but the actual decay rate corresponding to various Susy breaking spectra models (gravity mediation, gauge mediation, Higgs mediation etc) which are available in the literature may be used to constrain the predictions. Thus our analysis indicates that the road to a comprehensive numerical analysis of all aspects and predictions of the NMSGUT appears to be open. If the theory proves to generate consistent data points in spite of the nontrivial consistency check between fermion data and gauge unification then it will be an attractive contender for confronting the smoking gun data of Supersymmetry and proton decay discovery, if and when they arrive. This scenario amounts to an unfolding of the intricate(“ouroborotic”) interplay of the the tiny(neutrino) and superheavy mass scales which, in SO(10), are linked by the seesaw in the very guts of the NMSGUT.

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